ıs question paper contains 4 printed pages]

ır Roll No. : ....

No. of Q. Paper : 143 I

ıque Paper Code : 42354302

ıe of the Course : **B.Sc.(Prog.)/ B.Sc. Mathematical Sciences**

ıe of the Paper : Algebra

ıester : III

ıe : 3 Hours ***Maximum Marks : 75***

ıructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **Two** parts from each question.

(c) **All** questions are compulsory.

(d) **Marks** are indicated.

Unit- I

(a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R}; a \neq 0 \right\}$

Show that G is a group under matrix multiplication. 6

P.T.O.

(b) (i) Let G be a group such that if a, b, c and ab = ca ⇒ b = c, then prove tha is abelian.

(ii) Let $H = \{x \in U(20) : x \equiv 1 \bmod 3\}$.

List all elements of H.

Prove or disprove that H is a subgrou U(20).

(c) Prove that the intersection of two subgroup a group is a subgroup but their union is not.

2. (a) Define cyclic group. Prove that every cy group is Abelian. Is the converse tru Justify.

(b) Give an example of a non cyclic group al whose proper subgroups are cyclic.

(c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix}$

and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

(i) Write α and β as product of disj cycles.

(ii) Find $o(\alpha\beta)$ and $o(\alpha^{-1})$

3. (a) Let 'a' be an element of a finite grou Prove that $a^{o(G)} = e$.

(b) Consider the subgroup H = {1, 9} of group G = U(20) under multiplication modulo 20. Find the number of cosets of H in G and determine all the distinct cosets of H in G. 6

(c) Prove that the center Z (G) of a group G is a normal subgroup of G. 6

Unit- II

(a) Prove that a non empty subset S of a ring R is a subring of R if and only if

$a-b \in s$ and $ab \in S \; \forall \; a, b \in S$. 6.5

(b) Prove that $\mathbb{Q}\left[\sqrt{2}\right] = \left\{a + b\sqrt{2} : a, b \in \mathbb{Q}\right\}$ is an integral domain. 6.5

(c) (i) Let $\mathbb{Z}$ be the ring of integers and n be a fixed integer.

Show that $I = \langle n \rangle \{ nx : x \in \mathbb{Z} \}$ is an ideal of $\mathbb{Z}$. 3.5

(ii) Give an example of a finite , non commutative ring . 3

Unit- III

(a) Determine whether or not the set

$$\left\{\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right\}$$

is linearly independent over $\mathbb{Z}_5$. 6.5

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(b) Define the liner span of a subset of a vecto space V (F) and prove that the linear span of a set S is a subspace of V(F) containing S. 6.5

(c) Determine whether or not { (1, 3, 2), (2, 0, 1), (1, 1,1) } from a basis of R^3. 6.5

6. (a) Matrix of a linear transformation T with respect to basis {(1,2), (0,1)} of R^2 is given by $\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$.

Determine the linear transformation T. 6.5

(b) Let U and V be two finite dimensional vecto spaces over F. Let T from U to V be a linea transformation. If $\{u_1, u_2, u_3, \ldots\ldots\ldots\ldots, u$ generates U then show that Range space of T is generated by $\{T(u_1), T(u_2), T(u_3), \ldots\ldots\ldots\ldots, T(u_n)\}$. 6.

(c) Find the range, rank, kernel (Null space) an nullity of T where linear transformatio $T: R^2 \rightarrow R^3$ is defined by

$T(x, y) = (y, x + 2y, x + y)$.

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